



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Hays Lyle, an alumnus of Westminster College, a Theological student of Princeton Seminary for two years, and at present a minister in charge of a church at La Junta, Colorado.

Dr. Lyle, in 1884, married his second wife, Miss Mattie E. Grant, a scholarly and cultured lady, of Bardstown, Kentucky.

Dr. Lyle has been for many years an Elder in the Presbyterian Church, the church of his ancestors for, at least, the century and a half that have elapsed since his Great Grandfather emigrated from the northern part of Ireland to Berkeley County, Virginia.

THE CENTROID OF AREAS AND VOLUMES.

By G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

[Concluded.]

We will now find the centroid of the eighth part of the surface

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1, \text{ I, when } c=b, \text{ II, } c=a.$$

$$\text{We have } \bar{x} = \frac{\int x ds}{\int ds}, \quad \bar{y} = \frac{\int y ds}{\int ds}, \quad \bar{z} = \frac{\int z ds}{\int ds}.$$

$$\begin{aligned} \text{I. } s &= \frac{b}{a} \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} \left\{ \frac{a^4 - (a^2 - b^2)x^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} dx dy \\ &= \frac{\pi b}{2a^2} \int_0^a \sqrt{a^4 - (a^2 - b^2)x^2} dx = \frac{1}{4} \pi b \left(b + \frac{a}{e} \sin^{-1} e \right). \end{aligned}$$

$$s.\bar{x} = \int x ds = \frac{b}{a} \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} \left\{ \frac{a^4 - (a^2 - b^2)x^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} x dx dy$$

$$= \frac{\pi b}{2a^2} \int_0^a \sqrt{a^4 - (a^2 - b^2)x^2} \, x dx = \frac{\pi ab(a^2 + ab + b^2)}{6(a+b)}$$

$$\therefore \bar{x} = \frac{2a(a^2 + ab + b^2)}{3(a+b)(b + \frac{a}{e} \sin^{-1} e)}.$$

$$s.\bar{y} = s.\bar{z} = \int y ds = \frac{b}{a} \int_0^a \int_0^{\frac{a}{b} \sqrt{a^2 - x^2}} \left\{ \frac{a^4 - (a^2 - b^2)x^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} y dx dy$$

$$= \frac{b^2}{a^2} \int_0^a \sqrt{(a^2 - x^2)(a^2 - e^2 x^2)} dx = ab^2 \int_0^{\frac{1}{2}\pi} \sqrt{1 - e^2 \sin^2 \theta} \cos^2 \theta d\theta, \quad x = a \sin \theta$$

$$= \frac{ab^2}{3e^2} \left\{ (1 + e^2) E(e, \frac{\pi}{2}) - (1 - e^2) F(e, \frac{\pi}{2}) \right\}.$$

$$\therefore \bar{y} = \bar{z} = \frac{4ab \left\{ (1 + e^2) E(e, \frac{\pi}{2}) - (1 - e^2) F(e, \frac{\pi}{2}) \right\}}{3\pi e^2 (b + \frac{a}{e} \sin^{-1} e)}.$$

$$\text{II. } s = \frac{a}{b} \int_0^b \int_0^{\frac{a}{b} \sqrt{b^2 - y^2}} \left\{ \frac{b^4 + (a^2 - b^2)y^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} dy dx$$

$$= \frac{\pi a}{2b^2} \int_0^b \sqrt{b^4 + (a^2 - b^2)y^2} dy = \frac{\pi a^2}{4} \left\{ 1 + \frac{1 - e^2}{2e} \log \frac{1 + e}{1 - e} \right\}.$$

$$s.\bar{x} = s.\bar{z} = \int x ds = \frac{a}{b} \int_0^b \int_0^{\frac{a}{b} \sqrt{b^2 - y^2}} \left\{ \frac{b^4 + (a^2 - b^2)y^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} x dy dx.$$

$$s.\bar{x} = s.\bar{z} = \frac{a^2}{b^3} \int_0^b \sqrt{(b^2 - y^2)(b^4 + a^2 e^2 y^2)} dy$$

$$= a^2 \int_0^{\frac{1}{2}\pi} \sqrt{b^2 + a^2 e^2 \cos^2 \theta} \sin^2 \theta d\theta, \quad y = b \cos \theta$$

$$= a^3 \int_0^{\frac{1}{2}\pi} \sqrt{1 - e^2 \sin^2 \theta} \sin^2 \theta d\theta$$

$$= \frac{a^3}{3e^2} \left\{ (1-e^2)F(e, \frac{\pi}{2}) - (1-2e^2)E(e, \frac{\pi}{2}) \right\}.$$

$$\therefore \bar{x} = \bar{z} = \frac{4a \left\{ (1-e^2)F(e, \frac{\pi}{2}) - (1-2e^2)E(e, \frac{\pi}{2}) \right\}}{3\pi e^2 (1 + \frac{1-e^2}{2e} \log \frac{1+e}{1-e})}.$$

$$\begin{aligned} s.\bar{y} &= \frac{a}{b} \int_0^b \int_0^{\frac{a}{b} \sqrt{b^2-y^2}} \left\{ \frac{b^4 + (a^2-b^2)y^2}{a^2 b^2 - b^2 x^2 - a^2 y^2} \right\}^{\frac{1}{2}} y dy dx = \int y ds \\ &= \frac{\pi a}{2b^2} \int_0^b \sqrt{b^4 + a^2 e^2 y^2} y dy = \frac{\pi ab(a^2 + ab + b^2)}{6(a+b)}. \end{aligned}$$

$$\therefore \bar{y} = \frac{2b(a^2 + ab + b^2)}{3a(a+b)(1 + \frac{1-e^2}{2e} \log \frac{1+e}{1-e})}.$$

Since the limit of $\frac{\sin^{-1}e}{e}$ and $\frac{\log \frac{1+e}{1-e}}{2e}$ is 1 when $e=0$ we have, in either case, when $a=b$, $\bar{x}=\bar{y}=\bar{z}=\frac{1}{2}a$. The surface of the fourth part of the paraboloid $x^2+y^2=2a^2z$, for $z=h$.

$$s = \iint \sqrt{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dz}\right)^2} dz dx = \iint \sqrt{1 + \frac{x^2}{y^2} + \frac{a^4}{y^2}} dx dz.$$

$$\begin{aligned} \therefore s &= a \int_0^h \int_0^{a\sqrt{2z}} \sqrt{\frac{a^2+2z}{2a^2z-x^2}} dz dx = \frac{\pi a}{2} \int_0^h \sqrt{a^2+2z} dz \\ &= \frac{\pi a}{6} \left\{ (a^2+2h)^{\frac{3}{2}} - a^3 \right\}. \end{aligned}$$

$$\begin{aligned} s.\bar{x} &= s.\bar{y} = \int y ds = a \int_0^h \int_0^{a\sqrt{2z}} \sqrt{a^2+2z} dz dx = a^2 \int_0^h \sqrt{(a^2+2z)2z} dz \\ &= \frac{a^2}{16} \left\{ 2(a^2+4h)\sqrt{2a^2h+4h^2} - a^4 \log \left(\frac{a^2+4h+\sqrt{2a^2h+4h^2}}{a^2} \right) \right\}. \end{aligned}$$

$$\therefore \bar{x} = \bar{y} = \frac{3a \left\{ 2(a^2 + 4h) \sqrt{2a^2h + 4h^2} - a^4 \log \left(\frac{a^2 + 4h + 2\sqrt{2a^2h + 4h^2}}{a^2} \right) \right\}}{8\pi \{ (a^2 + 2h)^{\frac{3}{2}} - a^3 \}}.$$

$$\begin{aligned} s.z = \int z ds &= a \int_0^h \int_0^{a\sqrt{2z}} \sqrt{\frac{a^2 + 2z}{2a^2z - x^2}} z dz dx \\ &= \frac{\pi a}{2} \int_0^h \sqrt{a^2 + 2z} z dz = \frac{\pi a}{3} \left\{ (3h - a^2)(a^2 + 2h)^{\frac{3}{2}} + a^5 \right\}. \end{aligned}$$

$$\therefore \bar{z} = \frac{(3h - a^2)(a^2 + 2h)^{\frac{3}{2}} + a^5}{5 \{ (a^2 + 2h)^{\frac{3}{2}} - a^3 \}}.$$

The surface of the fourth part of the cone $x^2 + y^2 = a^2 z^2$, for $z = h$.

$$\begin{aligned} s &= \iint \sqrt{1 + \frac{x^2}{y^2} + \frac{a^4 z^2}{y^2}} dz dx = a \sqrt{1 + a^2} \int_0^h \int_0^{az} \frac{z dz dx}{\sqrt{a^2 z^2 - x^2}} \\ &= \frac{\pi a \sqrt{1 + a^2}}{2} \int_0^h z dz = \frac{\pi a h^2 \sqrt{1 + a^2}}{4}. \end{aligned}$$

$$s.x = s.y = \int y ds = a \sqrt{1 + a^2} \int_0^h \int_0^{az} z dz dx = a^2 \sqrt{1 + a^2} \int_0^h z^2 dx = \frac{a^3 h^3 \sqrt{1 + a^2}}{3}.$$

$$\therefore \bar{x} = \bar{y} = \frac{4ah}{3\pi}.$$

$$s.z = \int z ds = a \sqrt{1 + a^2} \int_0^h \int_0^{az} \frac{z^2 dz dx}{\sqrt{a^2 z^2 - x^2}} = \frac{\pi a \sqrt{1 + a^2}}{2} \int_0^h z^2 dz = \frac{\pi a h^3 \sqrt{1 + a^2}}{6}.$$

$$\therefore \bar{z} = \frac{2h}{3}.$$